

15/10/2026 - RECAP ON ROLE OF MPC INGREDIENTS + CASE STUDY ON BUILDING MANAGEMENT SYSTEM (BMS) (ECONOMIC MPC)

$$\min \left\{ \sum_{i=0}^{N-1} \underbrace{\ell(x_{i|k}, u_{i|k})}_{\text{STAGE COST}} + \underbrace{V_f(x_{N|k})}_{\text{TERMINAL COST}} \right\}$$

s.t.

$$\begin{cases} x_{i+1|k} = \underline{f}(x_{i|k}, u_{i|k}) ; i=0, 1, \dots, N-1 \\ x_{0|k} = x_k \end{cases}$$

$$\begin{cases} u_{i|k} \in \underline{U} ; i=0, 1, \dots, N-1 \end{cases}$$

$$\begin{cases} x_{i|k} \in \underline{X}, i=0, 1, \dots, N \end{cases}$$

$$x_{N|k} \in \underbrace{(X_f)}_{\text{TERMINAL SET}} \quad \text{TERMINAL CONSTRAINT}$$

ROLE OF N .

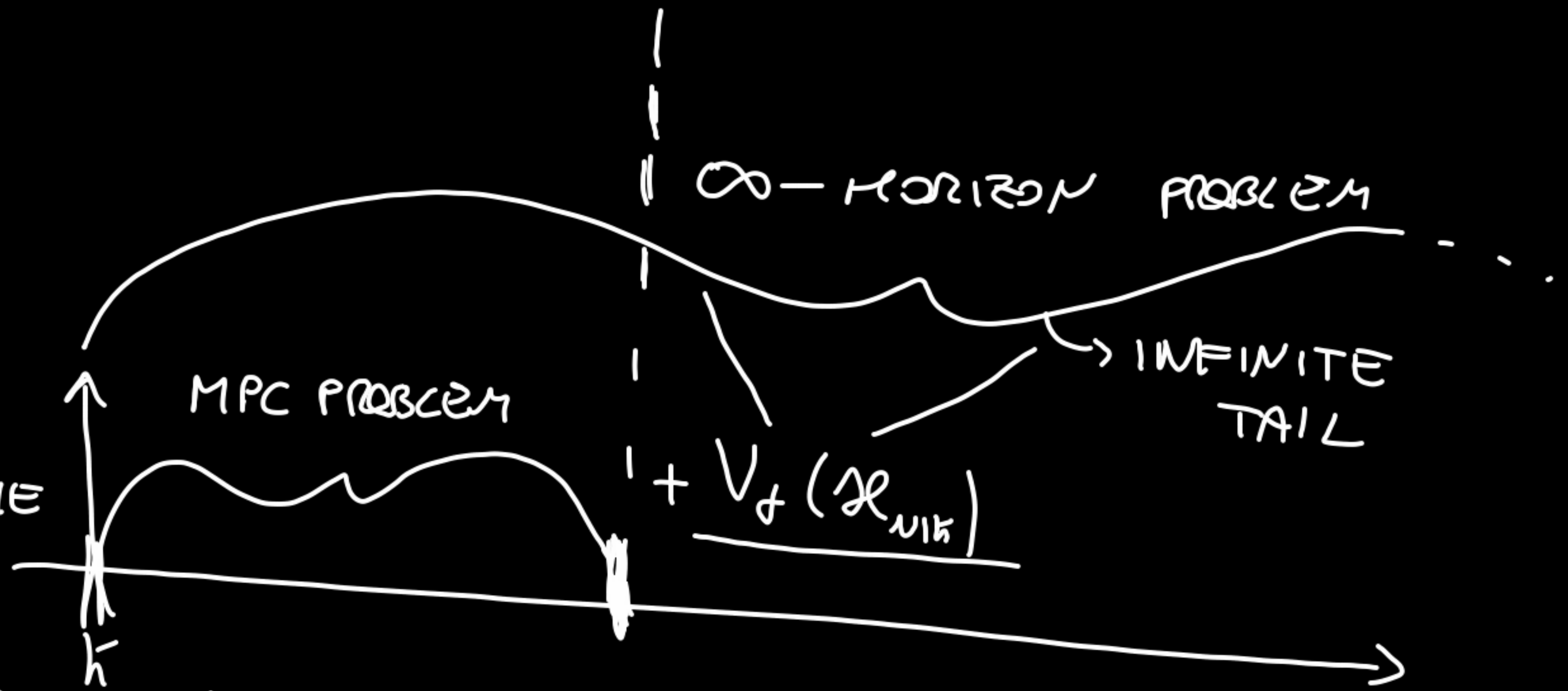
INFLUENCES:

- OPTIMALITY:

AS $N \rightarrow \infty$ MPC
APPROXIMATES THE
 ∞ -HORIZON

CONSTRAINED PROBLEM (WHICH GIVES THE BEST SOLUTION,
BUT THAT IS NOT POSSIBLE TO SOLVE IN GENERAL).

- COMPUTATIONAL COMPLEXITY: AS N INCREASES, THE SIZE
(NUMBER OF VARIABLES AND CONSTRAINTS GROWS) OF MPC
ITERATION GROWS $\rightarrow$ SOLVING TIME INCREASES.



ROLE OF $V_f(x_{N|k})$ - TERMINAL COST

IMPORTANT FOR:

① OPTIMALITY: IF WE CAN TAKE V_f AS THE VALUE FUNCTION OF THE ∞ -HORIZON PROBLEM (THE VALUE FUNCTION $V(x)$ GIVES THE COST OF OPTIMALLY CONTROLLING THE SYSTEM WHEN STARTING FROM INITIAL STATE x , FROM A GIVEN TIME TO INFINITY), THEN MPC IS OPTIMAL (MPC IS THEN AN ITERATIVE WAY TO SOLVE THE ∞ -HORIZON CONSTRAINED PROBLEM).

THIS DERIVES FROM THE BELLMAN OPTIMALITY PRINCIPLE.

IF WE CAN FIND ANY FEEDBACK CONTROLLER $\underline{u_k = C(x_k)}$
THAT STABILIZES $x^e = 0$, AND WE CAN PROVE
(1), (2) - WITH $\dot{V} \leq -\epsilon$
IT WITH LYAPUNOV, USING A LYAPUNOV FUNCTION

$V(x)$ $\rightarrow$ THEN, ALSO MPC, "TUNED" WITH
 $V_f \equiv \underline{V}$ WILL STABILIZE SYSTEM.

WHY? IF YOU DO THE PREVIOUS PROOF WITH

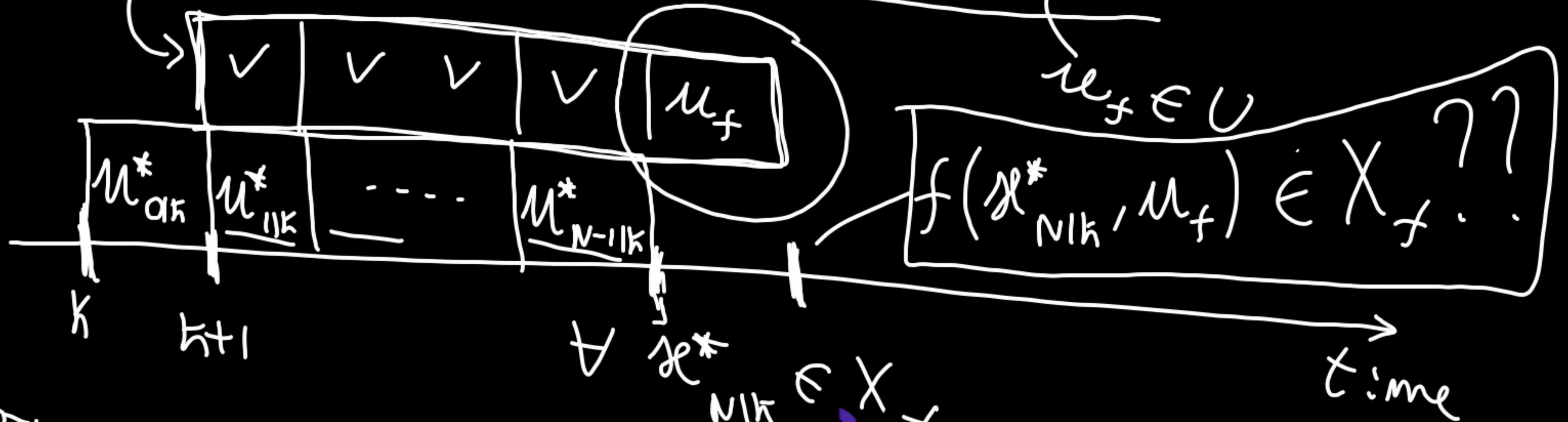
$V_f \equiv V$ AND $K_f \equiv C$, THEN $(*) \equiv (2)$, WHICH IS TRUE.

ROLE OF X_f (TERMINAL SET / TERMINAL CONSTRAINT)

CONNECTED TO
REC. FEASIBILITY

LET'S CONSIDER ANY GENERIC x_k : MPC IS FEASIBLE
($\forall x_k \in \mathcal{F}$), THEN CAN I SELECT INGREDIENTS
SUCH THAT $P_N(x_{k+1})$ IS

$$\hat{u}_{k+1} = \{u_{1|k}^*, u_{2|k}^*, \dots, u_{N-1|k}^*, \underbrace{u_f(x_{N|k}^*)}_{u_f \in U}\} \text{ FEASIBLE?}$$

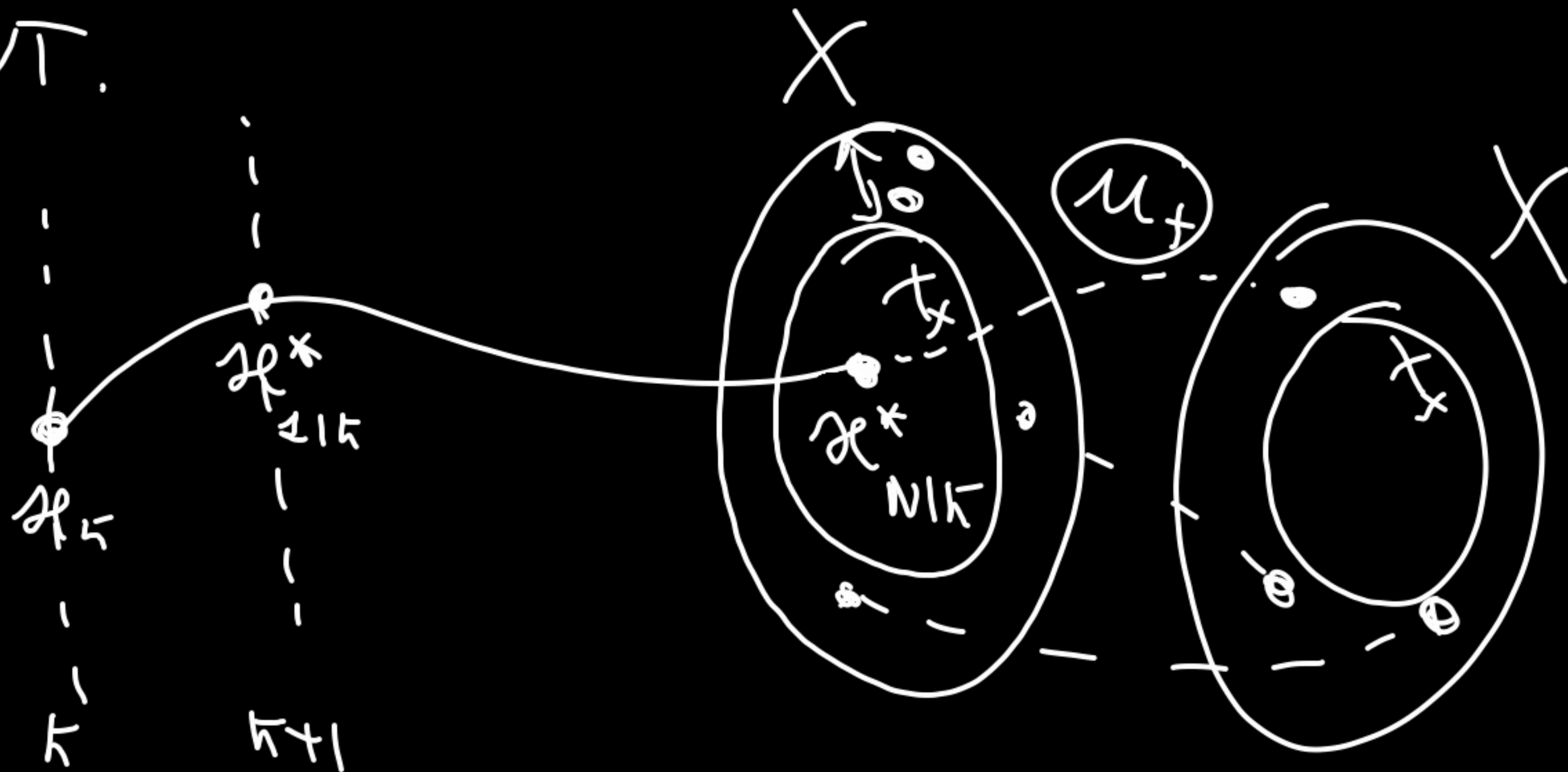


ASSUMPTION: NOMINAL CONDITIONS

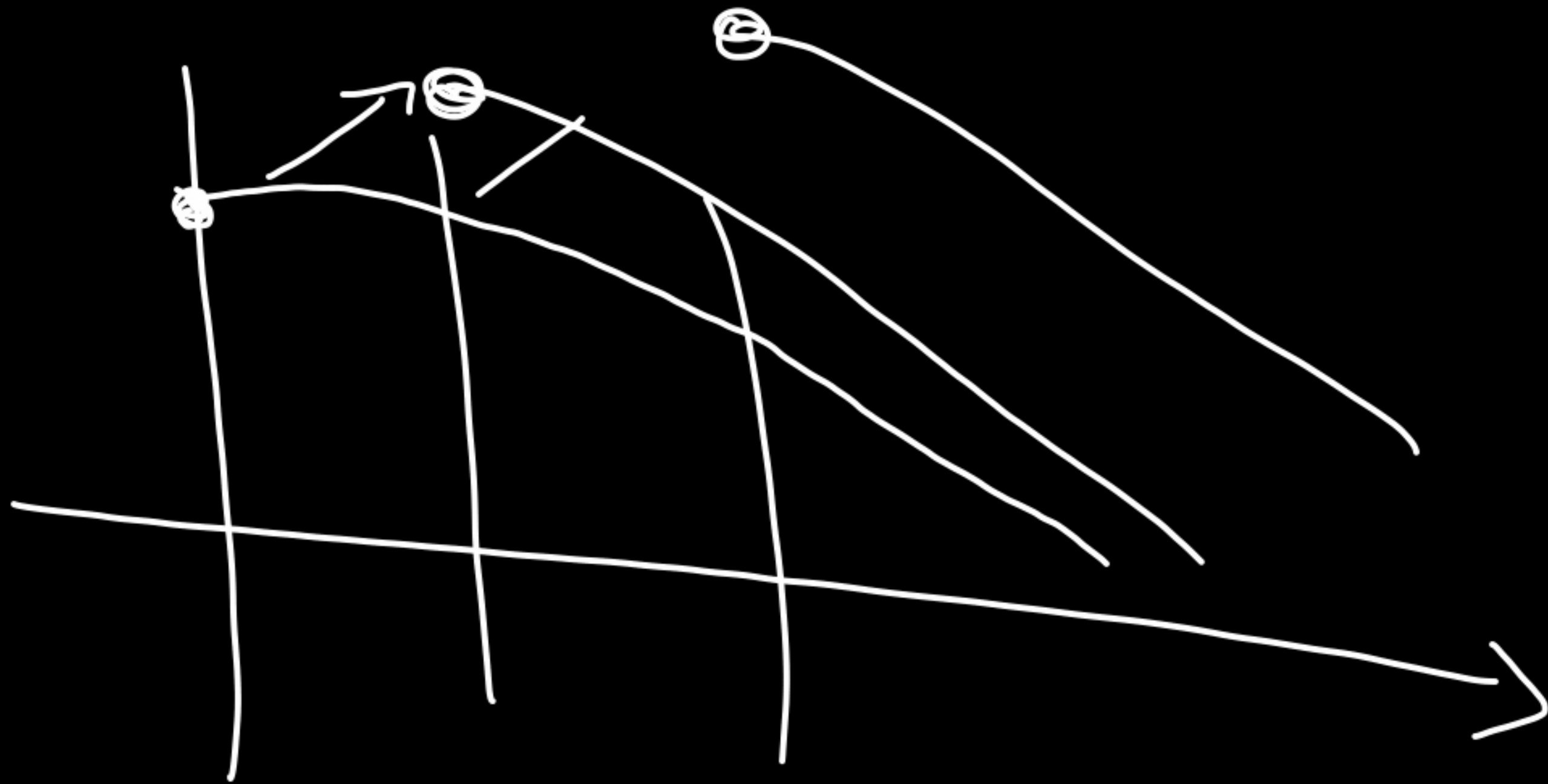
$$\forall x_{N|k}^* \in X_f$$

WE HAVE TO CHECK THAT $\forall x_{NIK}^*, u_f \equiv K_f(x_{NIK}^*)$ WILL KEEP
 NEXT STATE IN X_f ($f(x_{NIK}^*, u_f) \in X_f$).

THIS IS TRUE IF $u_f \equiv K_f(x)$ MAKES X_f CONTROLLED
 INVARIANT.



IDEALLY WE WOULD LIKE TO HAVE $X_f \equiv X$, SO THAT WE DON'T LOOSE PERFORMANCE. ($x_{nik} \in X_f \subset X$ IS AN "ARTIFICIAL" CONSTRAINT WE ADD TO GUARANTEE STABILITY)).

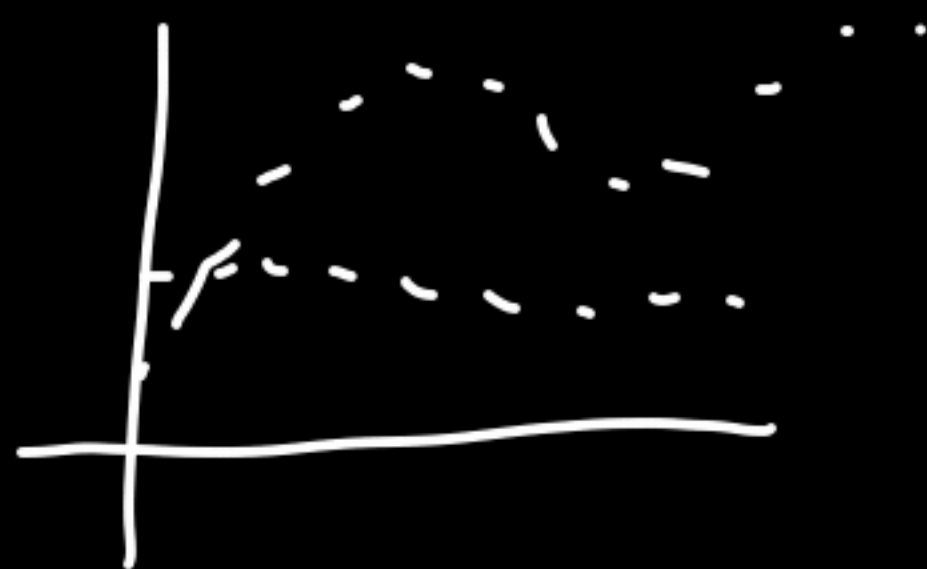


ECONOMIC MPC

CLASSIC (REGULATION/TRACKING MPC)

① GIVEN A
CONTROL
PROBLEM, IDENTIFY
CONTROL GOALS.

② TRANSLATE
GOALS INTO
REFERENCE
VALUES FOR
 y_k, u_k .



③ BUILD TRACKING/
REGULATION MPC

$$\left(\begin{array}{l} f(0,0) = 0 \\ \rho > 0 \\ V_f > 0 \end{array} \right) \rightarrow V_N > 0$$

ECONOMIC MPC (EMPC) APPROACH

① UNDERSTAND
CONTROL
GOALS

② WRITE THE
TARGET
FUNCTION IN
A WAY TO
CAPTURE / REPRESENT
DIRECTLY THE
GOALS AT POINT

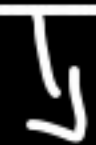
①

③ SOLVE EMPC



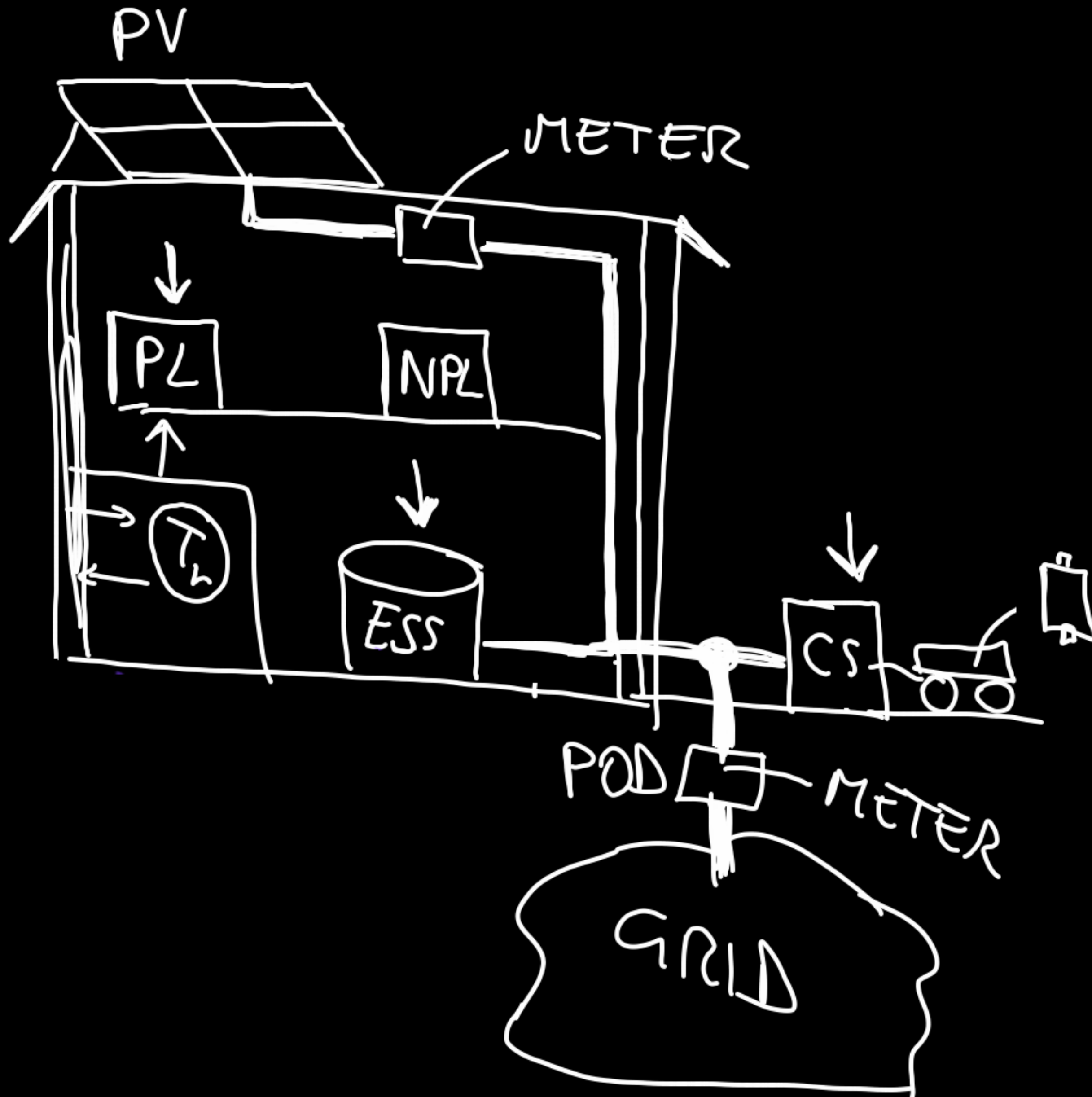
APPLY $u^*_{0:k}$

SOME OF PREVIOUS
ASSUMPTIONS
ARE LOST



PROPERTIES MORE
DIFFICULT TO STUDY

BMS CASE STUDY.



PV - PHOTOVOLTAIC PANELS

PL - PLANNABLE LOADS

NPL - NON-PLANNABLE LOADS

ESS - ENERGY STORAGE SYSTEM

CS - CHARGING STATION.

GOALS:

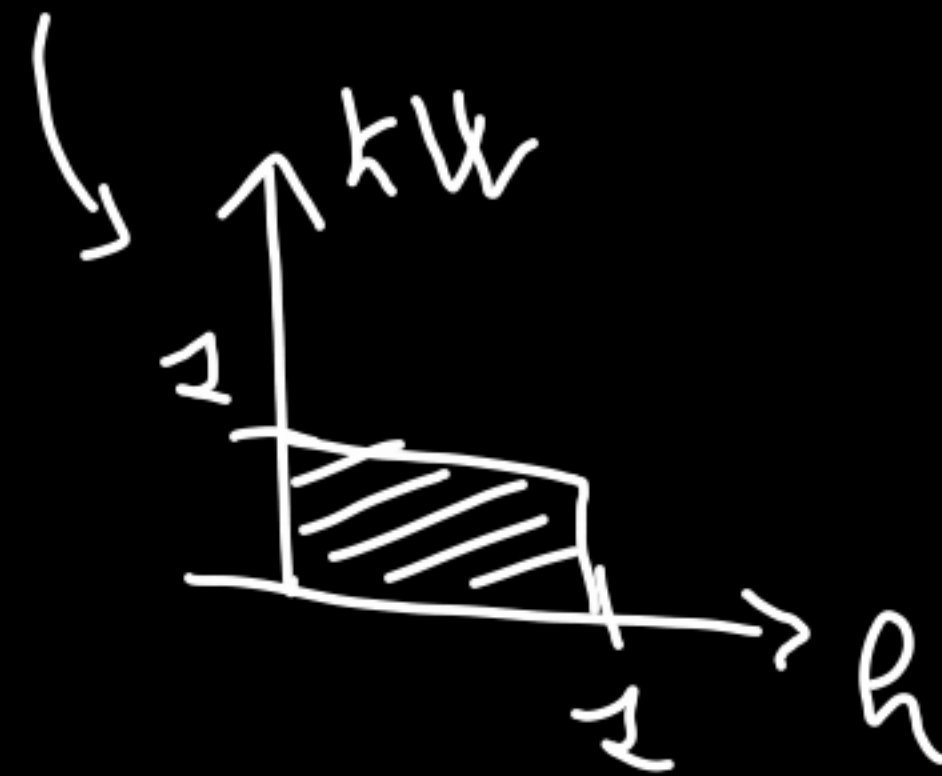
① MINIMIZE ELECTRICITY BILL

NOMENCLATURE

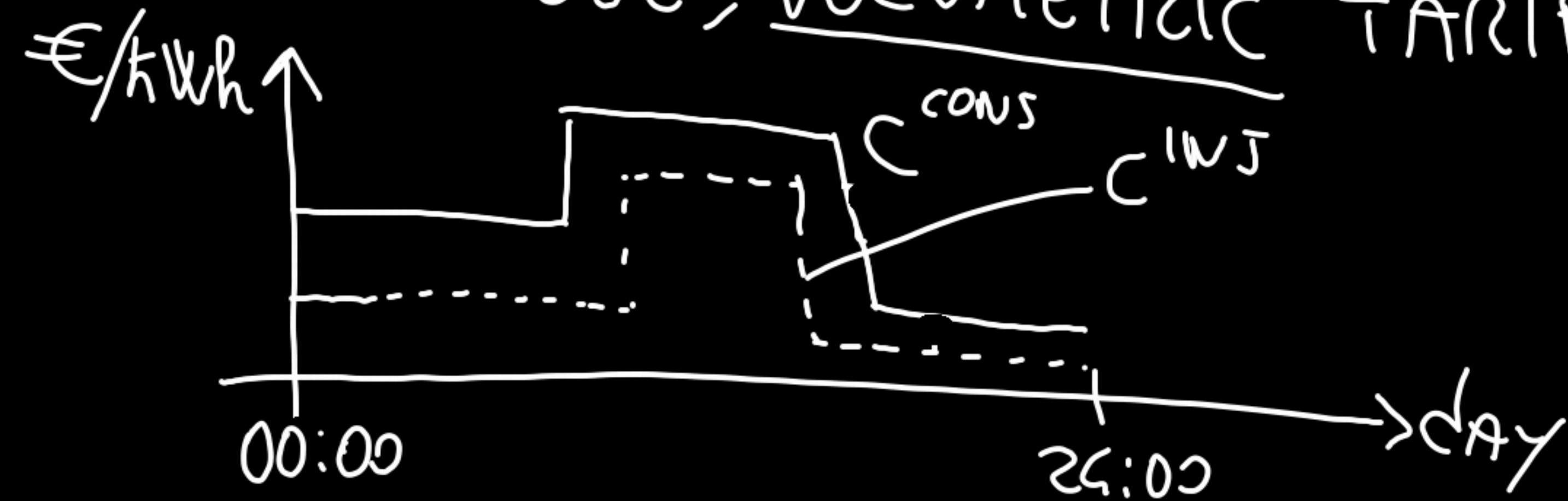
POWER VARIABLES [kW]: P^{PV} , P^{PL} , P^{NPL} , P^{ESS} , P^{PEV} , P^{POD}

ENERGY VARIABLES [kWh]
(STATE VARIABLES): x_k^{ESS} , x_k^{PEV}

ELECTRICITY BILL:



TIME OF USE, VOLUMETRIC TARIFF.



• C^{CONS} . COST OF CONSUMED ENERGY

• C^{INTJ} . REMUNERATION FOR INJECTED ENERGY

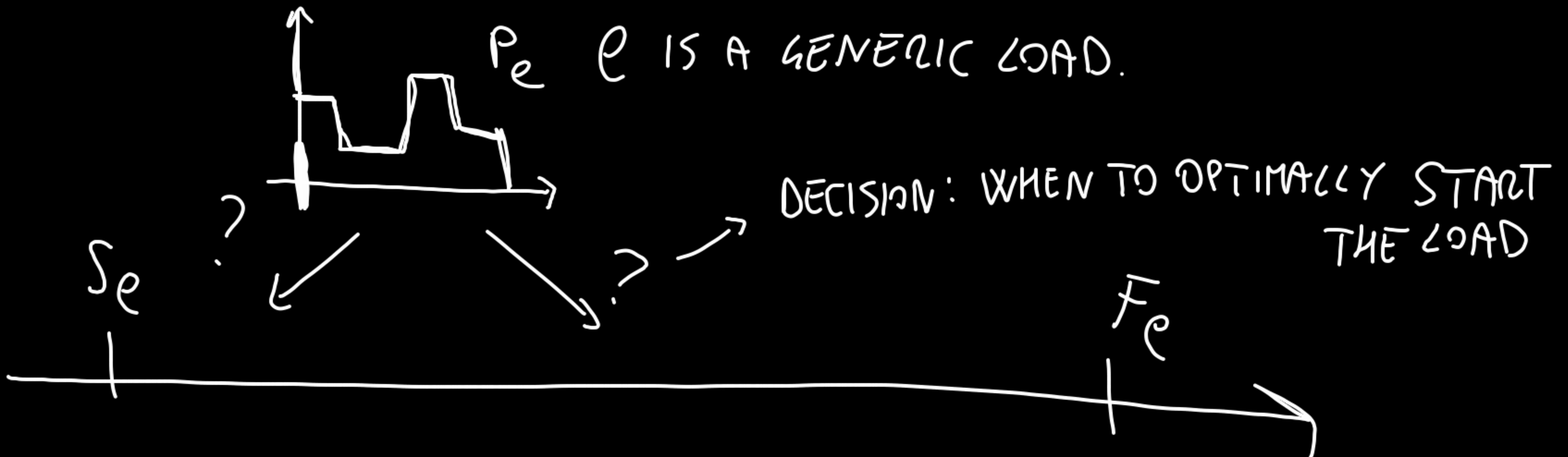
MAT. MODEL.

- $P^{PV} [kW]$ WE KNOW A PERFECT PREDICTION OF ITS VALUE IN THE FUTURE.
- $P^{NPL} [kW]$ SAME // . TOTAL CONSUMPTION OF NON PLANNABLE LOADS.
- P_{K}^{ESS} CONTROL VARIABLE (AS IN PAST LESSONS).
- P_{H}^{REV} CONTROL VARIABLE (// // //)

$P_k^{PL} [kW]$ IS THE TOTAL POWER CONSUMED AT TIME k BY THE PLANNABLE LOADS IN THE BUILDING.

TYPES OF PLANNABLE LOADS:

① LOADS THAT CAN BE SHIFTED IN TIME (LOAD SHIFTING).



② LOAD MODULATION : WE CAN CONTROL (MODULATE)

THE POWER CONSUMED BY THE LOAD.

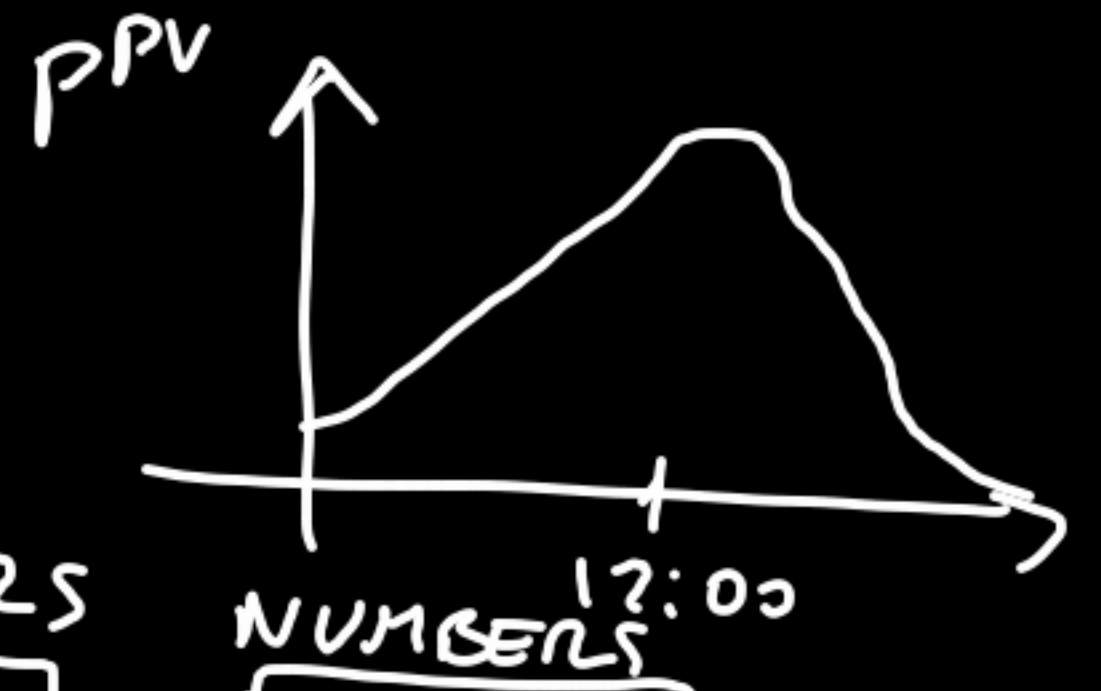
EXAMPLE : HVAC (HEATING, VENTILATION, AIR
CONDITIONING)



WE SKIP THIS PART

MPC FORMULATION

POWER BALANCE CONSTRAINT



- $$P_{i|k}^{POC} = P_{i|k}^{PEV} + P_{i|k}^{ESS} + P_{i|k}^{PL} + \overbrace{P_{i|k}^{NPL}}^{\text{NUMBERS}} - \overbrace{P_{i|k}^{PV}}^{\text{NUMBERS 12:00}} ; i=0,1,\dots,N-1$$

$\begin{pmatrix} +, - \\ \longleftrightarrow \end{pmatrix}$

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$\begin{pmatrix} + \\ \rightarrow \end{pmatrix}$

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$\begin{pmatrix} - \\ \leftarrow \end{pmatrix}$
- $$P_{i|k}^{ESS, ch} \geq 0, P_{i|k}^{ESS, dis} \geq 0 ; i=0,1,\dots,N-1$$
- $$P_{i|k}^{ESS} = P_{i|k}^{ESS, ch} - P_{i|k}^{ESS, dis} ; i=0,1,\dots,N-1 \quad \left(\begin{array}{l} \text{ESS POWER BALANCE} \\ \text{CONSTRAINT} \end{array} \right)$$

• $\delta_{i|h}^{ESS, ch}, \delta_{i|h}^{ESS, dis} \in \{0, 1\}; i=0, \dots, N-1$ (CHARGING/DISCHARGING INDICATOR VARIABLES)

• $p_{i|h}^{ESS, ch} \leq p^{ESS, MAX} \cdot \delta_{i|h}^{ESS, ch}; i=0, 1, \dots, N-1$ (CHARGING INDICATOR CONSTRAINT)

$$\left(\begin{array}{l} p_{i|h}^{ESS, ch} > 0 \longrightarrow \delta_{i|h}^{ch} = 1 \\ \text{(ESS CHARGING)} \\ \delta_{i|h}^{ch} = 0 \longrightarrow p_{i|h}^{ESS, ch} = 0 \end{array} \right)$$

• $p_{i|h}^{ESS, dis} \leq |p^{ESS, min}| \cdot \delta_{i|h}^{ESS, dis}, i=0, \dots, N-1$ (DISCHARGING INDICATION VARIABLE)

- $\delta_{ik}^{ESS, ch} + \delta_{ik}^{ESS, dis} \leq 1 ; i=0, 1, \dots, N-1$ (MUTUALLY EXCLUSIVE CHARGING/DISCH.)

AT ANY $i|k$, AT MOST ONE OF THE TWO δ CAN BE = TO 1 $\rightarrow$ AT LEAST ONE IS 0

$\rightarrow$ AT LEAST ONE BETWEEN p_{ik}^{ch} AND p_{ik}^{dis} IS 0. (BECAUSE OF PREVIOUS CONSTR.)

